Non linear dynamics in geosystems and reduced complexity models

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A tribute to
Claude Allègre
and
Jean-Louis Le Mouël

Advances in physics since the early twentieth century have shown how systems of coupled non-linear equations and reduced complexity models can effectively reproduce the dynamics of natural systems. Geophysical applications, originally proposed by Claude Allègre and Jean-Louis Le Mouël in IPGP, have multiplied in recent years, and these methods are now used routinely in many research laboratories, and have covered a wide range of phenomena, from the Deep Earth to Environmental systems and the Critical Zone. Here, based on these methods, we present an introduction to the concepts of complexity in geophysics and geochemistry.

The basics of the methods will be recalled for both M2 and ED students. Practice and applications will take place during the classes. Validation for M2 students will require participation and written exam. Validation for doctoral students will require participation and the detailed presentation (oral or written) of a case related to the subject of the thesis.

Course Sequence (revised)

Lecture 1 (FP) Jeudi 20 septembre 2018 14h-18h	**Universality of non-linear processes in geophysics at various scales.** From the Earth core to surface phenomena. From the laboratory to planets. Instabilities in underground systems, bifurcations, states and state jumps. Vorticity changes in fluids. Elementary non-linearities in physics and in geophysics: Boussinesq and Richards equations. Non-linear oscillators and relaxation oscillators. Relaxation oscillators in natural systems. The ideas of Yves Rocard: a pioneer of XXIst century geoscience. **The Van der Pol equation** and its properties. Solutions versus values of friction parameter. Analytical derivation of period, amplitude, period versus friction, and higher harmonic term.
Lecture 2 (FP) Jeudi 27 septembre 2018 16h-19h	**From linear to non-linear systems.** The Van der Pol oscillator with constant forcing. The forced Van der Pol oscillator. Mixed Modes Oscillations, quasiperiodicity, spiking, bursting, canards explosion and chaos. Non-linear stiffness: The Van-der-Pol Duffing equation and overview of properties. The stochastic Van der Pol equation. The Liénard equation and the Linéard graphical integration method. Non-Linear Differential systems. Autonomous systems. Introduction to the non-linear systems toolbox.

Lecture 3 (FP) Jeudi 4 octobre 2018 16h-19h	**From non-linearity to bifurcations.** A useful tool: revision of adaptive Runge-Kutta integration. Example with Rössler system. Revision of simple properties of two-dimensional linear systems. Conditions on the eigenvalues and Lyapunov factors. Example with non-linearized simple pendulum and a simple predator-prey model. Properties of normal modes: nodes, saddle and focus points. Trajectories in phase space and Poincaré sections. Dissipation in phase space. Existence and unicity of solutions: Cauchy-Lipschitz theorem. Example of non-unicity and blox up. One-dimensional non-lienarities and instabilities: normal forms and saddle-node, transcritical, supercritical and subcritical pitchfork bifurcations.
Lecture 4 (FP) Jeudi 11 octobre 2018 16h-18h	**From non-linearity to chaos.** Example of stabilizing term and hysteresis. Two-dimensional bifrucations. Example of two-dimensional saddle-node and supercritical. Hopf bifurcations: example of supercritical and subcritical Hopf bifurcations. One dimensional iterative maps. The logistic equation, the logistic map and its properties: period doubling and the road to chaos.
Lecture 5 (FP) Jeudi 18 octobre 2018 16h-18h	**Chaos and the Lorenz system.** Revisiting stability of limit cycles and two-dimensional bifurcations. Poincaré-Bendixson theorem. Fourier properties of the logistic time series. Sensitivity to initial conditions in the logistic equation. Chaos and sensitivity to initial conditions in the Hénon map. The Lorenz system. Dissipation and Lie derivative in the Lorenz system. Fixed points and stability. Breakdown of limit cycles. Poincaré sections and the lorenz map. Chaos and sensitivity to initial conditions in the Lorenz system.
Lecture 5 (FP) Jeudi 25 octobre 2018 16h-19h	**Complexity and non-linear dynamics in faults and earthquakes.** Scaling in faults and fractures. Earthquake models and Burridge-Knopoff non-linear system. Friction dynamics and heuristic models of the earthquake cycle.
Lecture 6 (CN) Jeudi 8 novembre 2018 16h-18h	**Introduction to the science of complex systems.** Presentation of the science of complex systems at the interface between dynamical systems theory and statistical mechanics. Complementarity between deterministic and stochastic models. What are the appropriate length and time scales for modeling? Presentation of reduced complexity models.
Lecture 7 (CN) Jeudi 15 novembre 2018 16h-18h	**Hierarchical systems in geophysics.** Critical phenomena and scale invariance. The origin of power laws in geophysics. The real-space renormalization group approach. Example 1: the temporal properties of seismicity. Example 2: Turbulence and geodynamo.
Lecture 8 (CN) Jeudi 22 novembre 2018 16h-18h	**Self-organization and the emergence of patterns.** Self-organization in non-equilibrium systems. Emergence of structures in geophysical systems.
Lecture 9 (CN) Jeudi 29 novembre 2018 16h-18h	**Sediment transport and geomorphic processes.** Sediment transport and surface flow. Landscape dynamics. Dune morphogenesis.
Lecture 10 (FP/CN) Jeudi 6 décembre 2018 16h-18h	Discussion and oral presentations.
xx janvier 2019 xxh-xxh	M2 written exam